Force Analysis of a Bio-inspired and Minimally Actuated Hopping Robot

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***Abstract*— The paper derives a mechanical model of a small and minimally actuated intermittent hopping robot by using the virtual work principle and the d' Alembert's principle. We analyse the characteristics of ground reacting forces and force-transforms of the robot by taking the friction under general condition into consideration. The paper also reports about hopping simulations and an experimental prototype. The analysis and experimental results show that the hopping robot, based on a non-symmetrical geared six-bar mechanism, can transform the linear force of spring to the nonlinear driving force of hopping motion and potentially overcome obstacles of 6~8 times of its body length by utilizing a single micro-motor as driving actuator. Furthermore, analysis results indicate that there can be the risk of losing mechanical energy by sliding friction if the anti-sliding measures are not effective enough.**

Keywords*— Robotics; Small Robot; Bio-inspired Robot; Hopping Robot; Jumping Robot

I. INTRODUCTION

There is an increasing interest in hopping or jumping robots for planetary surface exploration due to its flexible locomotion in rugged terrains and low/micro-gravity environments [1~3].

Various kinds of hopping systems[3~9] for planetary surface mobility have already been proposed. However, to meet the requirement of planetary mobility with minimal actuators and miniaturization, most of these paradigms are based on motion discontinuity, since a pause for reorientation and recharge of the thrust mechanism of paradigms inserted between jumps. A mostly mentioned work is the three generations hopping robots developed by Fiorini at al [4~6]. The first generation's hopping motion is directly generated by the release of a simple linear spring. The spring is compressed after each jump via a ball screw that is driven by a micro-motor. The hopping mechanism of the robot is compact and has only a single actuator, nevertheless, it is inefficient. To improve the energy conversion efficiency, the second and third generation, which are based on a symmetrical geared six-bar spring/linkage mechanism, can create a non-linear driving force from a linear spring that can overcome the shortcomings of the first generation system such as inefficiency and premature leaving the ground before full release of the spring. Although the hopping mechanism of the second and third generation is still a kind of simple ejection mechanism, it has motivated a new approach to efficiently convert stored energy to hopping motion by closed chain mechanism such as geared linkages.

Prior our work[10] has proposed a different paradigm (Fig. 1) based on a non-symmetrical geared six-bar mechanism. The driving force, driving torque of actuator, and the mechanical energy conversion efficiency of the robot were analysed by statics. However, the analysis results depend on the assumption that the ground static friction is under the critical status. In order to analyse the ground reacting force and the force-transforming characteristics of the proposed hopping robot by taking the actual ground friction in consideration, the mechanical model described in this paper is derived by using the virtual work principle and the d' Alembert's principle. The paper begins with deriving the mechanical model of the robot. Then, the force characteristics of the hopping mechanism of the robot are analysed, followed by hopping simulations and experiments of its prototype.

II. MECHANICAL MODELLING OF THE HOPPING ROBOT

Fig. 1 shows the mechanism model of the proposed bio-inspired and minimally actuated hopping robot[10]. The hopping mechanism is based on a non-symmetrical geared six-bar linkage, with a single degree of freedom (DOF) to realize biomimetic kangaroo-hopping posture and driving force.

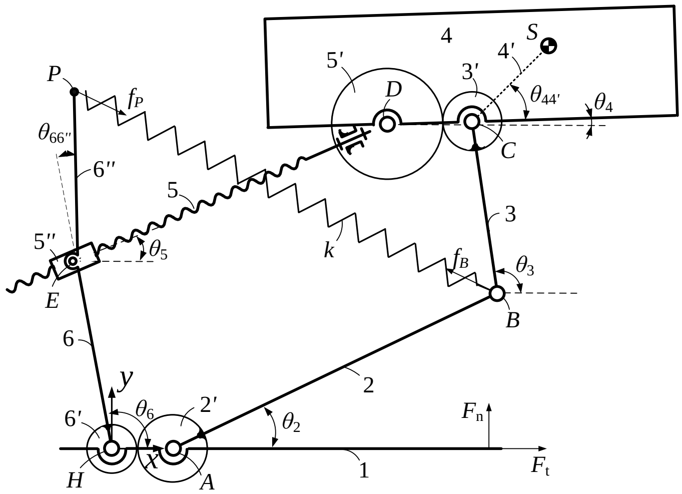

Fig. 1 Mechanism model of the bio-inspired hopping robot

By referring to the skeletal structure of kangaroo, the bar 1, 2, 3 and 4 represent its foot, shank, thigh and trunk respectively; the joint A, B and C represent its ankles, knee and hip respectively. Since the open chain linkage composed of the bar 1, 2, 3 and 4 is enclosed via 2 pairs of gears (2' and 6', 3' and 5'), bar 5 and 6, the non-symmetrical geared six-bar closed chain mechanism is obtained. The hopping mechanism takes extension spring as the energy storage/release component. In order to increase stored energy of the spring, the bar 6 is extended to bar 6'', and the both ends of the spring are separately fixed to the point P and joint B.

As showed in Fig. 1, the ground is taken as the frame of reference to establish the fixed-coordinate system, where the joint H is selected as the original point. Let li and θi represent the length and angle of bar i respectively, where l1 = lHA, l4 = lCD, θ1 = 0. According to the weight distribution characteristics of kangaroo, most of mass of the hopping mechanism is distributed on the trunk bar 1. Hence, we assumed bars except bar 1 (trunk) are lightweight that the total

mass of the robot is concentrated on the trunk and the mass of trunk is defined as m. Since the hopping mechanism has only a single DOF, let θ4 be the generalized coordinate. According to the constrains derived from the closed chain linkage

$$\left.\begin{aligned} l_1 + l_2 \mathrm{c}\theta_2 + l_3 \mathrm{c}\theta_3 - l_4 \mathrm{c}\theta_4 - l_5 \mathrm{c}\theta_5 - l_6 \mathrm{c}\theta_6 = 0 \\ l_2 \mathrm{s}\theta_2 + l_3 \mathrm{s}\theta_3 - l_4 \mathrm{s}\theta_4 - l_5 \mathrm{s}\theta_5 - l_6 \mathrm{s}\theta_6 = 0 \end{aligned}\right\} \quad (1)$$

where c $\triangleq$ cos, s $\triangleq$ sin, and the constrains derived from gear pairs

$$\left.\begin{aligned} \dot{\theta}_3 - \dot{\theta}_4 + i_1(\dot{\theta}_5 - \dot{\theta}_4) = 0 \\ \dot{\theta}_2 + i_2\dot{\theta}_6 = 0 \end{aligned}\right\} \quad (2)$$

where i1 is gear ratio of gear pair (3′ and 5′) and i2 is gear ratio of gear pair (2′ and 6′), the relation between q = [θ1, θ2, θ3, θ5, θ6]T and θ4 can be obtained as

$$\boldsymbol{q} = \boldsymbol{f}(\theta_4) \quad (3)$$

namely, each angle of bar i else can be expressed by θ4.

Differentiating Eq. (3), the constraints of motion of the hopping mechanism can be expressed by

$$\left.\begin{aligned} \dot{\boldsymbol{q}} &= J_C(\theta_4)\dot{\theta}_4 \\ \ddot{\boldsymbol{q}} &= J_C(\theta_4)\ddot{\theta}_4 + \dot{J}_C(\theta_4, \dot{\theta}_4)\dot{\theta}_4 \end{aligned}\right\} \quad (4)$$

where J_C is the Jacobean Matrix for $\dot{\boldsymbol{q}}$ and $\dot{\theta}_4$, and

$$J_C = \begin{bmatrix} \frac{\partial\theta_1}{\partial\theta_4} & \frac{\partial\theta_2}{\partial\theta_4} & \frac{\partial\theta_3}{\partial\theta_4} & \frac{\partial\theta_5}{\partial\theta_4} & \frac{\partial\theta_6}{\partial\theta_4} \end{bmatrix}^{\mathrm{T}} \quad (5)$$

Based on the configuration of the hopping mechanism, the position of the trunk's centre of mass (COM) S can be expressed by

$$\boldsymbol{r}_S = \begin{bmatrix} \sum_{j=1}^{3} l_j \cos\theta_j + l_{4'}\cos(\theta_{44'} + \theta_4) \\ \sum_{j=1}^{3} l_j \sin\theta_j + l_{4'}\sin(\theta_{44'} + \theta_4) \end{bmatrix} \quad (6)$$

where l4' = lCS, θ44' is the angle between the bar 4 and line CS.

By substituting Eq. (3) into Eq. (6), along with combining with Eq. (4), the first and second derivatives of Eq. (6), namely the velocity and acceleration of S, can be obtained as

$$\left.\begin{aligned} \dot{\boldsymbol{r}}_S &= \left[C_1(\theta_4)\dot{\theta}_4, C_2(\theta_4)\dot{\theta}_4\right]^{\mathrm{T}} \\ \ddot{\boldsymbol{r}}_S &= \begin{bmatrix} \dot{C}_1(\theta_4,\dot{\theta}_4)\dot{\theta}_4 + C_1(\theta_4)\ddot{\theta}_4 \\ \dot{C}_2(\theta_4,\dot{\theta}_4)\dot{\theta}_4 + C_2(\theta_4)\ddot{\theta}_4 \end{bmatrix} \end{aligned}\right\} \quad (7)$$

According to the general dynamics equations for system of particles, dynamics of the hopping mechanism can be expressed by

$$\sum_{i=1}^{n} \boldsymbol{Q}_i \bullet \delta\boldsymbol{r}_i = 0 \quad (8)$$

where Qi is the generalized positive forces acting on particle i and δri is the virtual displacement corresponding to Qi.

As shown in Fig 1, the generalized positive forces acting on the hopping mechanism contain inertia force F* and moment of inertia M* acting on S, spring tension force f and ground reacting force F. Further, f contains 2 component forces fP and fB acting on point P and joint B respectively, F contains 2 component forces, friction force Ft and normal supporting force Fn. The virtual displacements corresponding to F*, M*, fP, fB and F are δrS, δθ4, δrP, δrB and δrF.

$\boldsymbol{F}^*$ and $\boldsymbol{M}^*$ can be expressed by defining the acceleration of gravity and rotational inertia of trunk as $\boldsymbol{g}$ = [0, -g]$^{\mathrm{T}}$ and J respectively

$$\left.\begin{aligned} \boldsymbol{F}^* &= -m(\ddot{\boldsymbol{r}}_S + \boldsymbol{g}) \\ \boldsymbol{M}^* &= -J\ddot{\theta}_4 \end{aligned}\right\} \quad (9)$$

Likewise, fP and fB can be obtained as

$$\boldsymbol{f}_P = k(\|\boldsymbol{r}_B - \boldsymbol{r}_P\| - l_0)\frac{\boldsymbol{r}_B - \boldsymbol{r}_P}{\|\boldsymbol{r}_B - \boldsymbol{r}_P\|} = -\boldsymbol{f}_B \quad (10)$$

where k and l0 are stiffness coefficient and initial length of the spring respectively, θ66" is the angle between the bar 6 and bar 6", and

$$\boldsymbol{r}_P = [l_6\,\mathrm{c}\,\theta_6 + l_{6''}\,\mathrm{c}(\theta_6 - \theta_{66''}), l_6\mathrm{s}\theta_6 + l_{6''}\mathrm{s}(\theta_6 - \theta_{66''})]^{\mathrm{T}}$$
$$\boldsymbol{r}_B = [l_1 + l_2\,\mathrm{c}\,\theta_2, l_2\mathrm{s}\theta_2]^{\mathrm{T}}$$

Noting that f is a kind of internal force, while the external forces are only F. Hence, Fn can be expressed by using the d' Alembert's principle.

$$\boldsymbol{F}_{\mathrm{n}} = -\boldsymbol{F}^* \bullet \boldsymbol{e}_y \quad (11)$$

where e is the unit vector that ex = [1, 0]T, ey = [0, 1]T.

Since $\boldsymbol{F}_{\mathrm{t}}$ has such 2 probable statuses such as static friction force and sliding friction force. Hence, defining the displacement of the bar 1 (foot) as x_d while $\boldsymbol{F}_{\mathrm{t}}$ is sliding friction force, $\boldsymbol{F}_{\mathrm{t}}$ can be obtained as

$$F_{\mathrm{t}} = \begin{cases} -\boldsymbol{F}^* \bullet \boldsymbol{e}_x & F_{\mathrm{t}} \le u_s F_{\mathrm{n}} \\ -\mu_d F_{\mathrm{n}} \mathrm{sign}(\dot{x}_d) & F_{\mathrm{t}} > u_s F_{\mathrm{n}} \end{cases} \quad (12)$$

where μs is the coefficient of static friction, μd is the coefficient of sliding friction.

Since the bar 1 (foot) keeps contact to the ground during the hopping robot's takeoff phase, depending on the status of $\boldsymbol{F}_{\mathrm{t}}$, $\delta\boldsymbol{r}_{\mathrm{F}}$ corresponding to $\boldsymbol{F}$ can be expressed as

$$\delta\boldsymbol{r}_{\mathrm{F}} = \begin{cases} [0, 0]^{\mathrm{T}} & F_{\mathrm{t}} \le u_s F_{\mathrm{n}} \\ [\delta x_d, 0]^{\mathrm{T}} & F_{\mathrm{t}} > u_s F_{\mathrm{n}} \end{cases} \quad (13)$$

Substituting Eq. (9) ~ (13) into Eq. (8), mechanical model of the hopping mechanism can be obtained as

$$\boldsymbol{F}^* \bullet \delta\boldsymbol{r}_S + \boldsymbol{M}^* \bullet \delta\theta_4 + \boldsymbol{f}_P \bullet \delta\boldsymbol{r}_P + \boldsymbol{f}_B \bullet \delta\boldsymbol{r}_B + \boldsymbol{F} \bullet \delta\boldsymbol{r}_F = 0 \quad (14)$$

where F = [Ft, Fn]T.

III. CALCULATION RESULTS

According to the kangaroo's physical configuration and the hopping robot's own characteristics, the parameters of the hopping robot are specified as listed in Table 1.

TABLE I
PARAMETERS OF THE ROBOT

Item	Value	Item	Value	Item	Value
l_1 /mm	25	l_6 /mm	80.9	m /kg	1.3
l_2 /mm	160	$l_{6''}$ /mm	70	J/kg·m^2	0.0046
l_3 /mm	80	$l_{4'}$ /mm	40	k /(N/m)	400
l_4 /mm	35	$\theta_{44'}$ /(°)	10	i_1	0.28
l_5 /mm	155.9	$\theta_{66''}$ /(°)	15	i_2	6.37

Substituting initial value $[\theta_4, \boldsymbol{q}]^T$ = [0º, 0º, 10º, 165º, 14.8º, 173.85º]^T, μ_s = 1.0, μ_d = 0.7 and the parameters in Table 1 in Eq. (11) ~ (14). Define $F_t = \mu_s F_n$ as the critical value of static frication and F_n = 0 as the end condition of calculation respectively. F_t and F_n during the hopping robot's takeoff phase can be calculated as shown in Fig. 2.

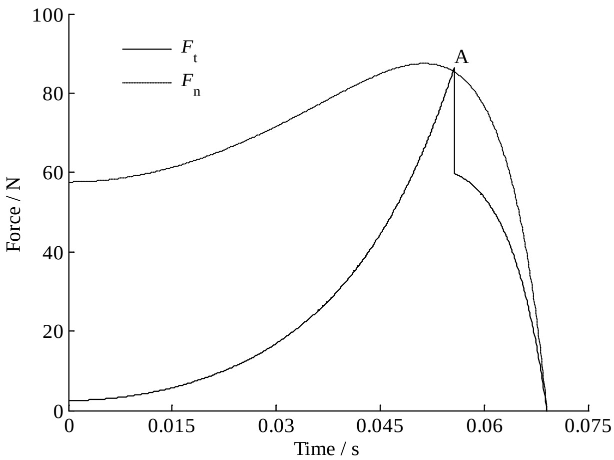

Fig. 2 F_t and F_n during takeoff phase

As shown in Fig. 2, F_t and F_n are low at the onset and then increase smoothly till the final part of the takeoff phase; while the spring force reaches maximum at the onset of takeoff and then decreases. The non-linear characteristic of F_t and F_n is in agreement with the law of the ground reacting force acting on kangaroo during its takeoff phase[11], which indicates that this hopping mechanism has biomimetic hopping capabilities that can convert the linear spring tension force to the nonlinear driving force. Although F_t is lower than F_n at the onset, it increases rapidly with time passes. Around the late-mid period of the takeoff phase, F_t catches up with F_n and then steeply declines at the point **A** as show in Fig 2, which shows F_t reaches its critical status at the point **A** followed by turning into sliding friction force from static friction force. μ_d is assumed as 1.0 that is a relative large value of coefficient of sliding friction, however, the sliding still exists in the calculation results. Hence, there is the risk to loss mechanical energy by sliding friction if μ_d is not large enough.

The hopping mechanism is a spring-linkage system whose driving force is converted from the tension force of spring, namely, the spring tension force $\boldsymbol{f}$ is the original input force of the hopping mechanism. According to the mechanical model, the ground reacting force $\boldsymbol{F}$ is the driving force to generate hopping motion due to the fact that it is the only external force. Hence, the force-transforming characteristics of the hopping mechanism can be described by the ratio of $\boldsymbol{F}$ versus $\boldsymbol{f}$, namely the mechanical advantage η of the hopping mechanism. According to calculation results of Eq. (14), η can be obtained as shown in Fig 3.

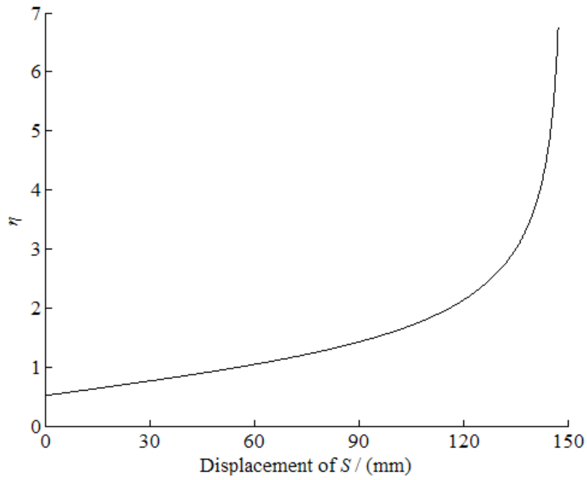

Fig. 3 The mechanical advantage of the hopping mechanism

As shown in Fig. 3, η is lower than 1.0 in the initial period of takeoff phase that shows the driving force is relatively low though the spring force is high. Along with the release of the spring, the COM of trunk S moves forward and upward, meanwhile, η increases. This increase of η causes the gain of the driving force. Hence the driving force increases continuously though the spring force decreases gradually. This indicates the hopping mechanism can buffer the vigorous spring force at the onset and gain the decreasing spring force in late-mid period, which prevents the hopping robot from premature leaving the ground before full release of the spring and increases the efficiency of mechanical energy conversion to obtain larger hopping distance. Further, since the peak value of the driving force is reduced by comparing with spring-actuated directly, the rated torque of the hopping robot's actuator is reduced. This indicates that a smaller power actuator can recharge the hopping robot's energy storage mechanism.

IV. SIMULATIONS AND EXPERIMENTS

A. *Hopping Simulations*

Based on the analysis mentioned above, the 3D model of the hopping robot is built and the dynamic simulations of hopping process are carried out by using ADMAS. Fig. 4 presents the ground reacting force that is measured by simulations during the hopping robot's takeoff phase. The curves of the ground reacting force are substantially in agreement with the calculation results as shown in Fig. 2, which further proves the biomimetic characteristics of the hopping robot.

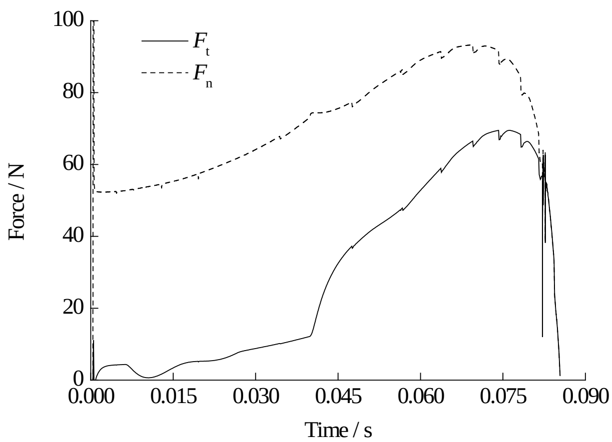

Fig. 4 Ground reacting force measured by hopping simulations.

B. Experiments

The prototype was designed as shown in Fig. 5. To reduce the output power of the power supply, the serial control strategy is also adopted in addition to using micro-motor as actuator. That is to say, there is only one motor working in each phase of hopping process as shown in Fig. 6.

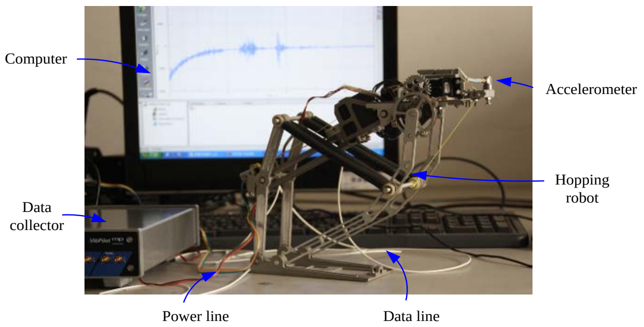

Fig. 5 Prototype of the proposed hopping robot

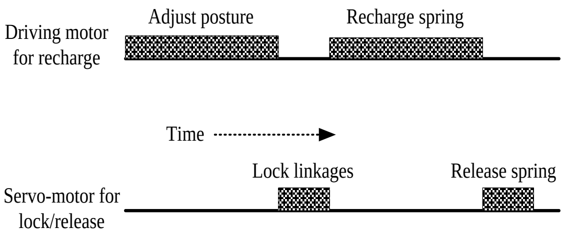

Fig. 6 Control sequence of micro-motors used in the prototype

The hopping experiments were carried out in indoor environment. Fig. 7 presents the acceleration of the robot's trunk that is measured by using accelerometers. The hopping robot begins with recharging the spring for hopping and its acceleration is correspondingly flat curve. Then, along with the release of the spring, the prototype takes off and its acceleration rapidly increases. When the hopping robot is about to depart off the ground, the acceleration reaches its maximum. Following the lift-off, the acceleration falls into minimum and then appears vibratory. This is because the hopping robot needs to pull its foot (bar 4) off the ground in short time and the spring will shake as it is close to original length. The time of hopping is very short. However, as shown in the enlarged view (a) in Fig. 7, the convex profile of acceleration during takeoff is in agreement with the simulation results as shown in Fig. 4, which verified the availability of analysis.

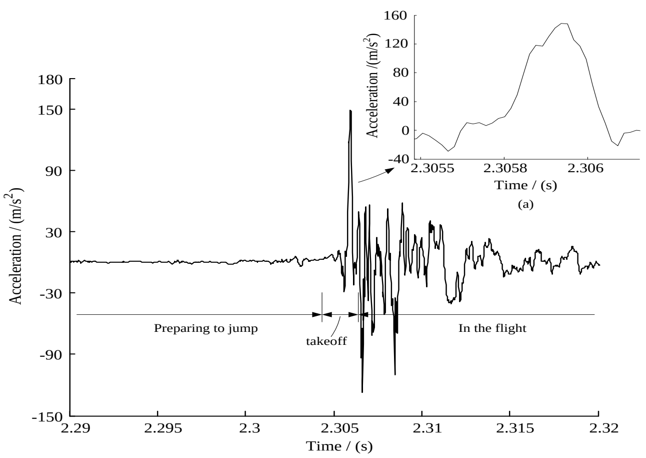

Fig. 7 The experimental results of the acceleration of the robot's trunk, where the sub-graph (a) details the trunk's acceleration during takeoff

The prototype of the hopping robot weighs only 0.62kg with its control system excluded. It typically hops a horizontal distance of 1.60 ~ 1.75m and reaches a vertical height of 0.3 ~ 0.45m, which demonstrates that the robot can potentially overcome obstacles of 6~8 times of its body length. Fig. 8 shows the snapshots of experimental hopping phase, which was filmed by the high speed camera system, "MotionBLITZ Cube", with 120 frames per second.

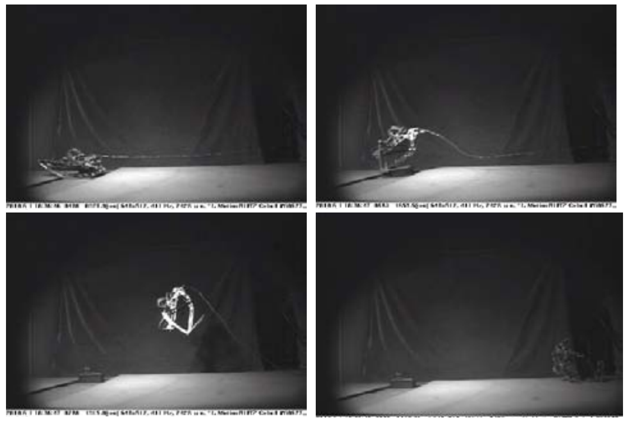

Fig. 8 Video snapshots of hopping experiments

The small sliding of the prototype's foot can be found out from the snapshots although the experiments are conducted on a rough wooden board. This verifies the risk of losing mechanical energy by sliding friction if the anti-sliding measures are not effective enough.

V. CONCLUSIONS

The conclusions are summarized as follows:

1) Mechanical model of a small bio-inspired hopping robot is derived from using the virtual work principle and the d' Alembert's principle. The calculation, simulation and experiment results show that the hopping robot based on a non-symmetrical geared six-bar mechanism can transform the linear force of the spring to the nonlinear driving force of hopping motion and potentially overcome obstacles of 6 ~ 8 times of its body length by utilizing a single micro-motor as driving actuator.

2) The friction involved in the mechanical model of the proposed robot is under general condition. Analysis results indicate that there is a risk of losing mechanical energy caused by sliding friction if the anti-sliding measures are not effective enough.

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